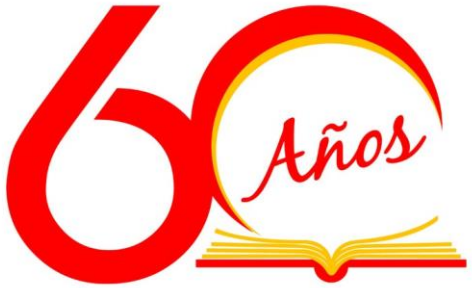
Álgebra



CICLO REPASO UNI

ACADEMIA CÉSAR VALLEJO

Teoría de funciones I

CICLO REPASO UNI

FUNCIÓN

Sean A y B conjuntos no vacíos. La función f de A en B es un conjunto de pares ordenados $(x; y)$ tal que a $x \in A$ le corresponde un único elemento $y \in B$.

Notación: $f : A \rightarrow B$

$x \rightarrow y = f(x)$

Dominio de f

Esta dada por la variación de x, si esta no se conoce entonces por lo general:

$$\text{Dom } f = A \cap (CVA \text{ de } f(x))$$

Para hallar el CVA consideramos:

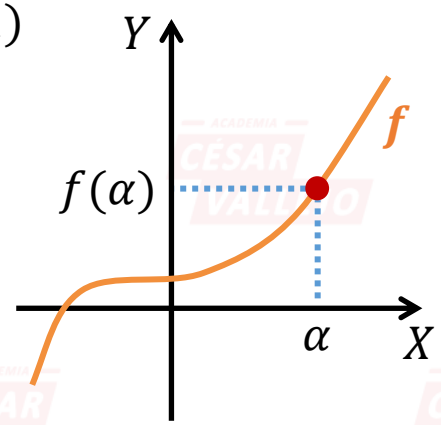
$\sqrt[\text{par}]{a} \in \mathbb{R} \Leftrightarrow a \geq 0$	$\frac{P_{(x)}}{Q_{(x)}} \in \mathbb{R} \Leftrightarrow Q_{(x)} \neq 0$

Rango de f

Esta dada por la variación de $f(x)$, y se obtiene a partir del dominio.

GRÁFICA DE FUNCIONES

1)



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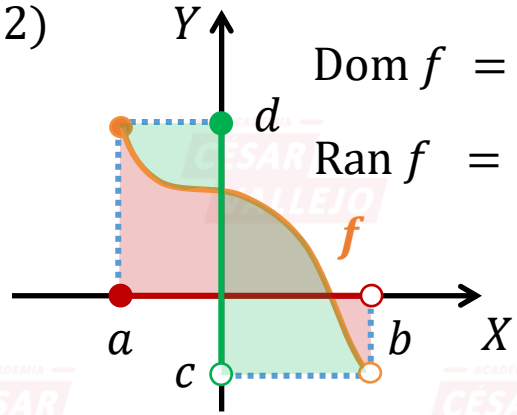
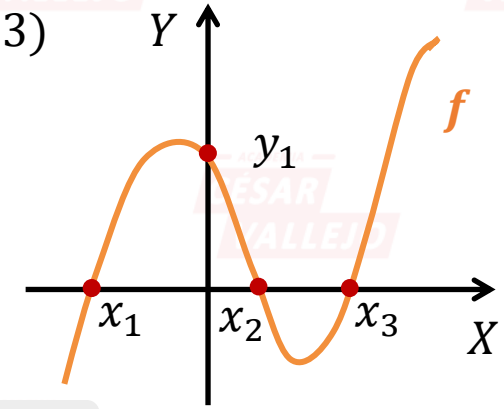


$\text{Dom } f = [a; b\rangle$

$\text{Ran } f = \langle c; d]$

3)



Corte con el eje Y

$y_1 = f(0)$

Corte con el eje X

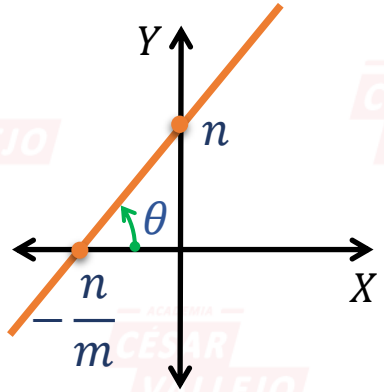
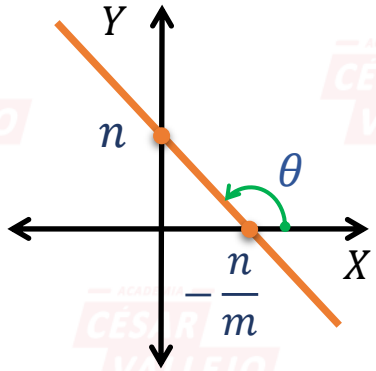
$f(x) = 0$

su CS $= \{x_1; x_2; x_3\}$

Sean A y B conjuntos no vacíos. La función f de A en B es un conjunto de pares ordenados $(x; y)$ tal que a $x \in A$ le corresponde un único elemento $y \in B$.

FUNCIÓN LINEAL

$f(x) = mx + n$ $\quad m \neq 0$

$m > 0$	$m < 0$
	

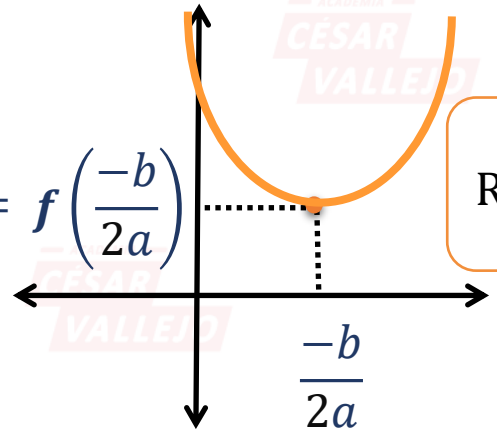
Pendiente $\quad m = tan\,\theta$

FUNCIÓN CUADRÁTICA

$f(x) = ax^2 + bx + c$ $\quad a \neq 0$

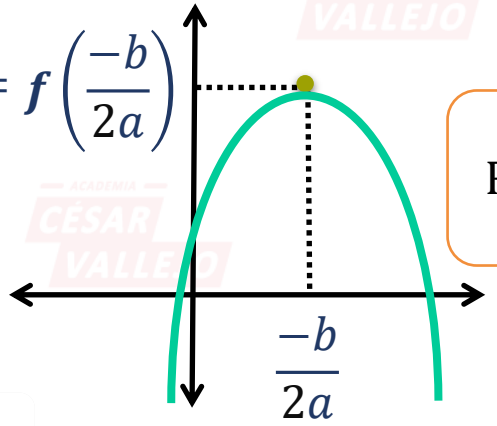
Grafiquemos considerando dos casos:

I) $a > 0$ Parábola cóncava hacia arriba



$$\text{Ran}f = \left[f\left(\frac{-b}{2a}\right); +\infty\right\rangle$$

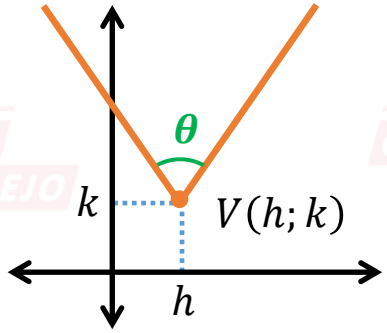
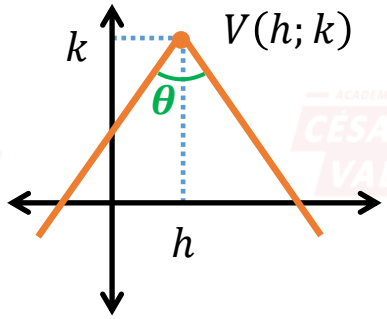
II) $a < 0$ Parábola cóncava hacia abajo



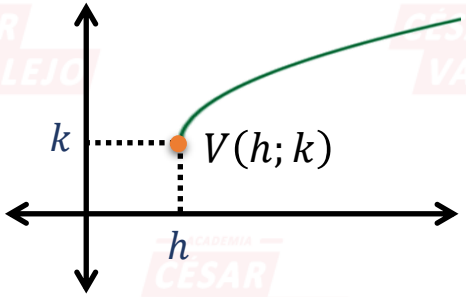
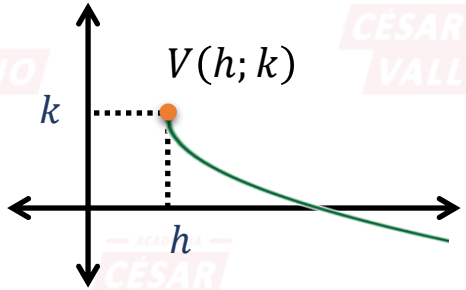
$$\text{Ran}f = \left\langle -\infty; f\left(\frac{-b}{2a}\right)\right]$$

FUNCIÓN VALOR ABSOLUTO Y RAIZ CUADRADA

$$f(x) = a|x - h| + k \qquad a \neq 0$$

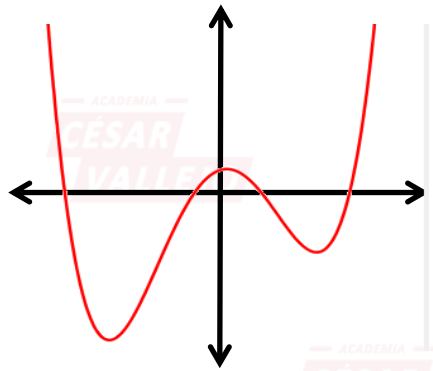
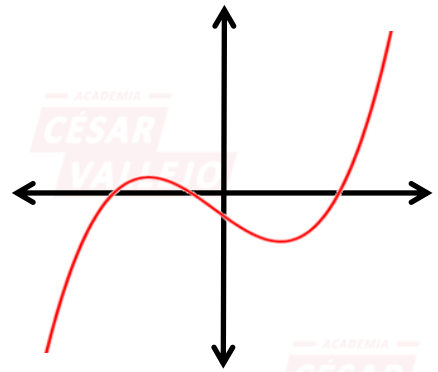
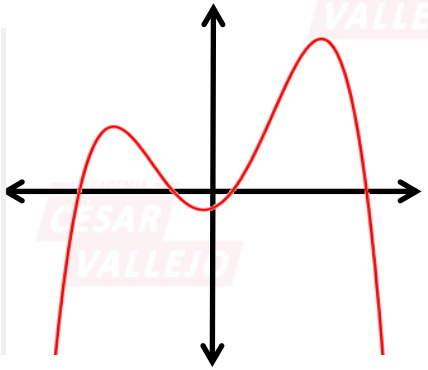
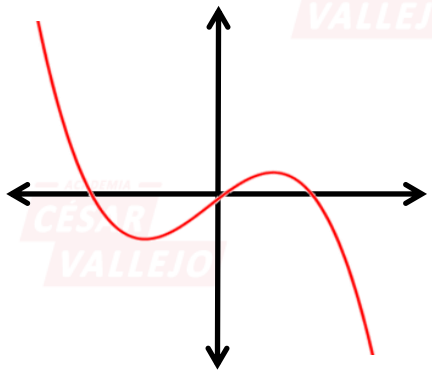
Si $a > 0$	Si $a < 0$
	

$$f(x) = a\sqrt{x - h} + k \qquad a \neq 0$$

$a > 0$	$a < 0$
	

FUNCIÓN POLINOMIAL DE GRADO n

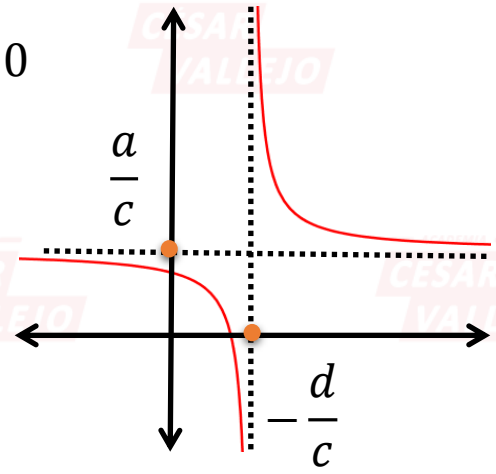
$$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$$

	n es par	n es impar
$a_0 > 0$		
$a_0 < 0$		

FUNCIÓN FRACCIONARIA

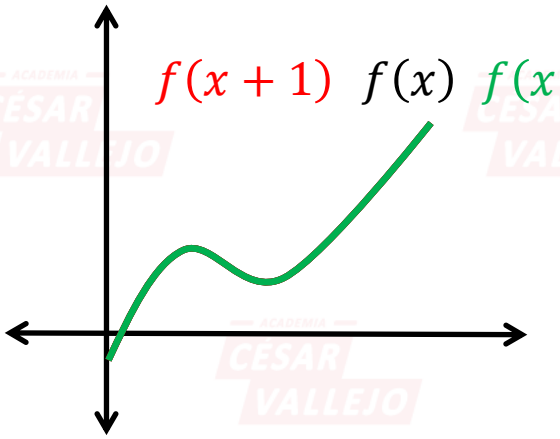
$$f(x)=\frac{ax+b}{cx+d} \qquad c\neq 0$$

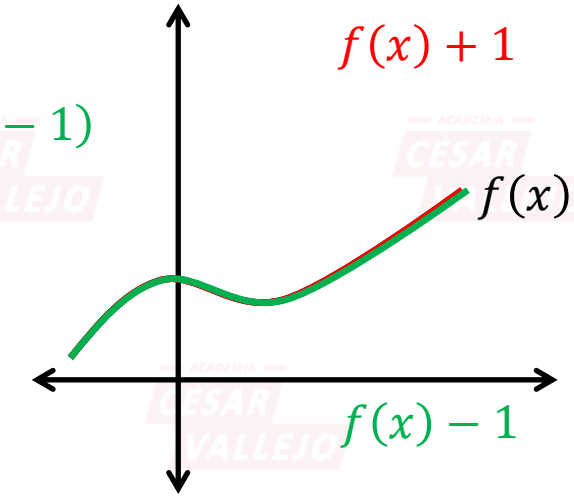
Su gráfica es una hipérbola de asíntotas.

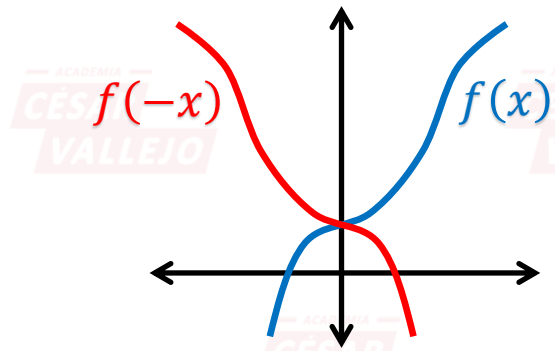


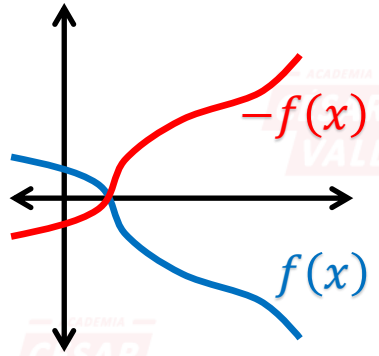
$$y=\frac{a}{c} \qquad x=-\frac{d}{c}$$

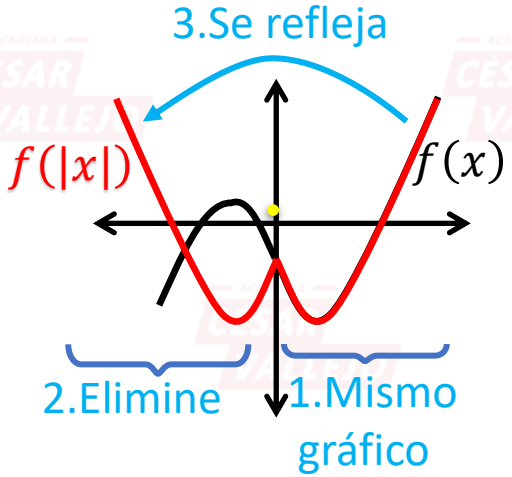
PROPIEDAD DE GRÁFICAS

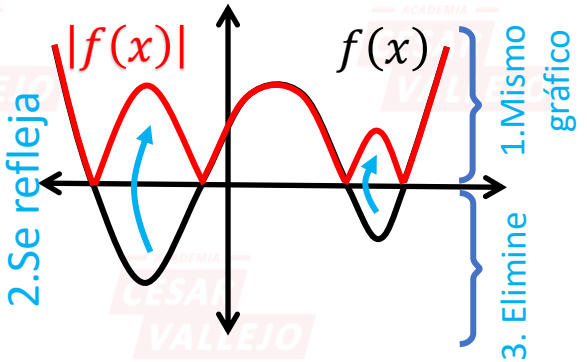












FUNCIÓN CRECIENTE

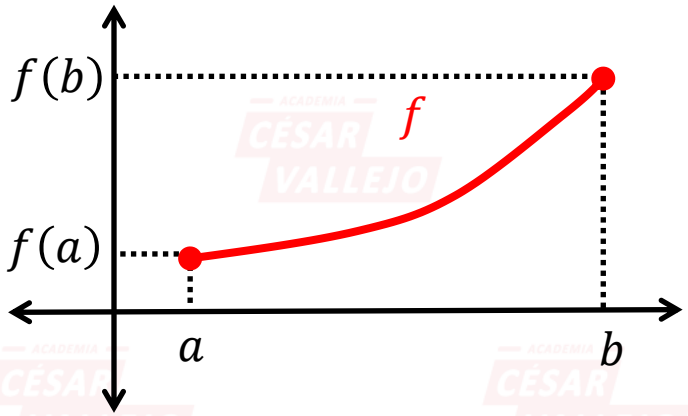
La función $\boldsymbol{f}$ es creciente en el intervalo $\boldsymbol{I}$ de su dominio si para todo par de números $\boldsymbol{x_1}$ y $\boldsymbol{x_2}$ de dicho intervalo se cumple

$$x_1 < x_2 \rightarrow f(x_1) < f(x_2)$$

Teorema:

Sea f una función continua y creciente.

Si Dom $f = [a; b]$ entonces Ran $f = [f(a); f(b)]$



FUNCIÓN DECRECIENTE

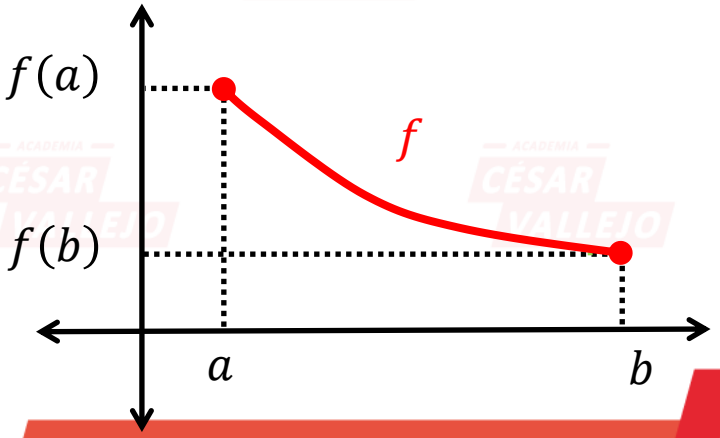
La función $\boldsymbol{f}$ es decreciente en el intervalo $\boldsymbol{I}$ de su dominio si para todo par de números $\boldsymbol{x_1}$ y $\boldsymbol{x_2}$ de dicho intervalo se cumple

$$x_1 < x_2 \rightarrow f(x_1) > f(x_2)$$

Teorema:

Sea $\boldsymbol{f}$ una función continua y decreciente.

Si Dom $f = [a; b]$ entonces Ran $f = [f(b); f(a)]$



ÁLGEBRA DE FUNCIONES

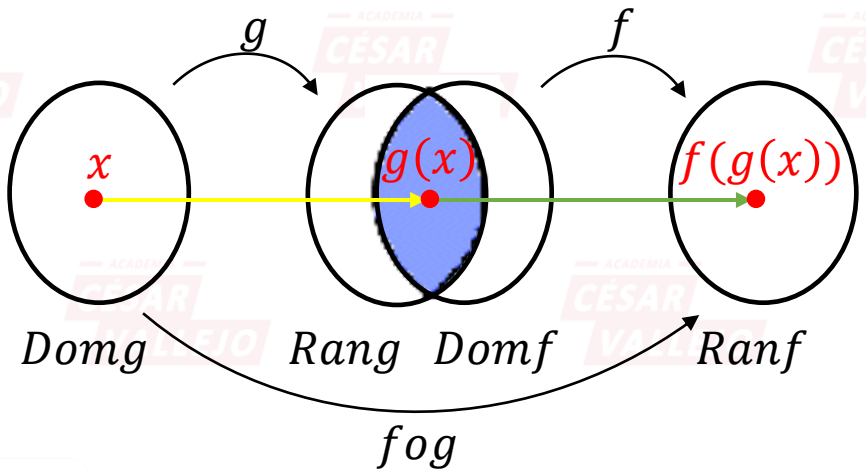
Notación	Regla de correspondencia Dominio
$f+g$	$(f+g)(x)=f(x)+g(x)$ $Dom(f+g)=Dom(f)\cap Dom(g)$
$f-g$	$(f-g)(x)=f(x)-g(x)$ $Dom(f-g)=Dom(f)\cap Dom(g)$
$f.g$	$(f.g)(x)=f(x).g(x)$ $Dom(f\cdot g)=Dom(f)\cap Dom(g)$
$\frac{f}{g}$	$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}$ $Dom(f)\cap Dom(g)-\{x/g(x)=0\}$
f^n	$f^n(x)=[f(x)]^n$ $Dom\,(f^n)=Domf$

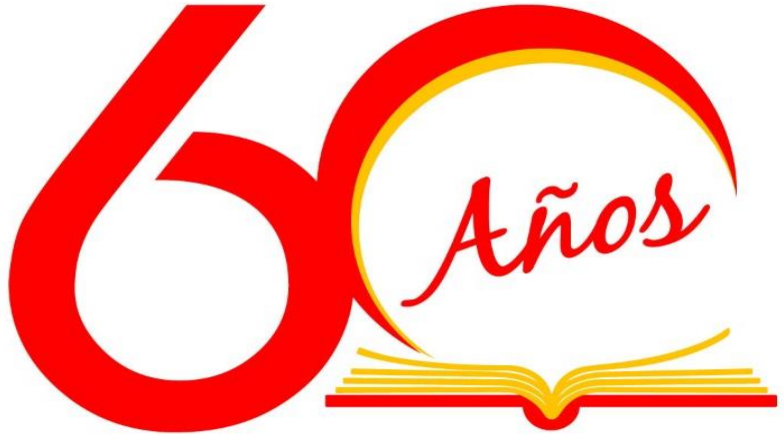
COMPOSICIÓN DE FUNCIONES

Sean f y g son dos funciones, se denota y define su composición (f compuesta con g) por :

$$fog\begin{cases} Dom(fog)=\{x/x\in Dom\ g\ ;\ g(x)\in Domf\} \\ (fog)(x)\ \ =f(g(x)) \end{cases}$$

En forma gráfica, tenemos:





¡Gracias!